

Uncoupled Nuclear Matter Approximation*

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A relatively simple nuclear matter approximation, based on a result derived by Masterson and Sawada, is evaluated. In this approximation free kinetic energies are used in the energy denominators of the reaction matrices instead of the self-consistent energies, and a second-order calculation of the true momentum density is incorporated into the equations for the system energy. If the density is computed directly with a separable potential, the equations of the theory are uncoupled. One evaluation of the new approximation, using a modification of the code used by Brueckner and Masterson, yielded an equilibrium mean energy of -14.0 MeV at 1.04 F, in good agreement with Brueckner and Gammel's -15.2 MeV at 1.02 F and with Brueckner and Masterson's -16.9 MeV at 1.00 F. All the calculations referred to employed the same hard-core phenomenological potential. A further calculation is planned to compute with precision the Green's functions and momentum densities required for the straightforward application of this approximation. When completed and reported, these quantities will make the remainder of the nuclear matter calculation quite simple. Several possible future uses of the approximation are briefly discussed.

I. INTRODUCTION

IN a series of previous papers by Brueckner and collaborators,¹ methods have been developed for the determination of the properties of nuclear matter. Accurate numerical solutions of the equations of the theory gave a mean binding energy of -15.2 MeV and equilibrium spacing of 1.02 F for one of the sets of Gammel-Thaler potentials,² slightly modified to give correctly the low-energy scattering parameters and the deuteron properties. These values were in reasonably good agreement with the semiempirical values for the energy ranging from -15.83 reported by Green³ to -17.04 obtained by Cameron,⁴ and with the equilibrium spacing, $r_0 = (1.07 \pm 0.02)$ F, deduced from high-energy electron-nucleus scattering. A detailed discussion of Brueckner K -matrix theory and the method of applying it to extended nuclear matter is contained in the paper by Brueckner and Gammel¹ (BG). Consequently, we shall indicate here only the basic equations.

In the Brueckner-Gammel computations, the K matrix from which the energy is computed is defined by the equation

$$K_{ij;kl} = v_{ij;kl} + \sum_{mn} v_{ij;mn} \frac{(1-n_m)(1-n_n)}{\omega_k + \omega_l - \omega_m^* - \omega_n^*} K_{mn;kl}. \quad (1.1)$$

In this equation ω_k and ω_l are self-consistent energies

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¹ K. A. Brueckner and J. L. Gammel, Phys. Rev. **109**, 1023 (1958). Referred to as BG. This paper gives extensive references to previous work. See also, K. A. Brueckner, *The Many-Body Problem*, edited by C. DeWitt and P. Nozières (John Wiley & Sons, Inc., New York, 1959), p. 47; *Quantum Theory*, edited by D. R. Bates (Academic Press Inc., New York, 1962), p. 286.

² J. L. Gammel and R. M. Thaler, Phys. Rev. **107**, 291 (1957); **107**, 1337 (1957).

³ A. E. S. Green, Rev. Mod. Phys. **30**, 569 (1958); Phys. Rev. **95**, 1006 (1956).

⁴ A. G. W. Cameron, Can. J. Phys. **35**, 1021 (1957).

for particles moving in the Fermi gas and ω_m^* and ω_n^* are energies appropriate to virtual excitations above the Fermi surface. The single-particle potential is determined from the diagonal elements of the K matrix by the relation

$$V(p_i) = \sum_j (K_{ij;ij} - K_{ij;ji}) n_j = \sum_j K_{ij;(ij)} n_j, \quad (1.2)$$

where n_j is the occupation number for states of momentum p_j (and for zero-temperature nuclear matter is the Fermi step function). The average binding energy per particle is

$$E_{av} = \frac{3}{p_F^3} \int_0^{p_F} p^2 dp \left[\frac{p^2}{2m} + \frac{1}{2} V(p) \right]. \quad (1.3)$$

The normal density is assumed to be determined from the minimum of E_{av} as a function of density.

In a later paper, Brueckner and Masterson⁵ (BM) reported the results of the computationally simpler approximation which results when the difference of energies in the denominator of Eq. (1.1) is assumed to be independent of the total momentum. Thus, they made the replacement

$$\omega_k + \omega_l - \omega_m^* - \omega_n^* = 2[\omega(p_{kl}) - \omega^*(p_{mn})], \quad (1.4)$$

with p_{kl} and p_{mn} the relative momenta. This approximation is accurate if ω_k has a quadratic dependence on p_k or if the relative momentum is large compared with the total momentum. Consistent with the accuracy of this approximation, BM replaced the total momentum (which enters in the treatment of the exclusion principle) by its average value compatible with a given value of relative momentum k :

$$\frac{\langle P^2 \rangle_{av}}{4} = \frac{\frac{3}{5} p_F^2 \left(1 - \frac{k}{p_F} \right) \left(1 + \frac{1}{2} k/p_F + \frac{1}{6} k^2/p_F^2 \right)}{\left(1 + \frac{1}{2} k/p_F \right)}, \quad k \leq p_F. \quad (1.5)$$

For $k > p_F$, $\bar{P} = (\langle P^2 \rangle_{av})^{1/2} = 0$.

⁵ K. A. Brueckner and K. S. Masterson, Jr., Phys. Rev. **128**, 2267 (1962). Referred to as BM.

The Brueckner-Masterson approximation also yielded good agreement with experiment, the mean binding energy being -16.9 MeV at $r_0=1.00$ F.

In this paper a further approximation is suggested and evaluated. Motivated by a result derived by Masterson and Sawada (MS),⁶ this approximation features a release from the "self-consistency" requirement which complicates other nuclear matter calculations. There are only two computationally difficult quantities required, the Green's functions and the momentum density (through second order). The former, being independent of the nucleon-nucleon potential in this approximation, can be computed once (using an electronic computer) and tabulated for all future use of the approximation. The momentum densities can also be calculated (with a separable potential) and tabulated. The remaining equations are easily handled for typical nucleon phenomenological potentials such as, e.g., the Gammel-Thaler potentials.

II. THE UNCOUPLED APPROXIMATION

Starting with the Brueckner-Goldstone expansion⁷

$$\begin{aligned} E &= \sum_k \epsilon_k n_k + \Delta E(n, \epsilon) \\ &= \sum_k \epsilon_k n_k + \frac{1}{2} \sum_{kl} v_{kl; (kl)} n_k n_l \\ &\quad + \lim_{\alpha \rightarrow 0} \frac{1}{4} \sum_{kl, mn} v_{kl; (mn)} \frac{(1-n_m)(1-n_n)n_k n_l}{(\epsilon_k + \epsilon_l - \epsilon_m - \epsilon_n + i\alpha)} \\ &\quad \times v_{mn; (kl)} + O(v^3) \end{aligned} \quad (2.1)$$

(where the ϵ are the free kinetic energies and the n the expectation values of the number operator), MS were able to demonstrate, with a "change-of-parameter" technique, that

$$E = \sum_k \epsilon_k n_k + [\Delta E(\rho, \epsilon)]_{\text{no self-energy processes}} \quad (2.2)$$

is exact to fourth order.⁸ In this equation, the "self-energy" terms of the Brueckner-Goldstone expansion are neglected and the n_k appearing in the expansion are replaced by the true momentum densities in the interacting medium, ρ_k . Equation (2.2) leads to an expansion matrix

$$t_{ij; kl'} = v_{ij; kl} + \sum_{mn} v_{ij; mn} \frac{(1-\rho_m)(1-\rho_n)}{\epsilon_k + \epsilon_l - \epsilon_m - \epsilon_n} t_{mn; kl'} \quad (2.3)$$

so that

$$E = \sum_k \epsilon_k n_k + \sum_{kl} t_{kl; (kl)}' \rho_k \rho_l. \quad (2.4)$$

In the absence of an experimental ρ_k , the ρ_k in Eqs.

⁶ K. S. Masterson, Jr., and K. Sawada, Phys. Rev. **133**, A1234 (1964). Referred to as MS.

⁷ K. A. Brueckner, Phys. Rev. **100**, 36 (1955); J. Goldstone, Proc. Roy. Soc. (London) **A239**, 267 (1957).

⁸ Equation (2.2) is exact to fourth order in the single-particle potential v . Further, if Eq. (2.1) is expanded in terms of any t matrix [such as Eq. (2.3)], then Eq. (2.2) is exact to fourth order in t .

(2.3) and (2.4) would be very difficult to obtain because they are related to the t matrices in a self-consistent way. Consequently, we are forced to make further approximations. One of these is to use the expectation values n_m and n_n in the t -matrix equation in place of the momentum densities, and the other is to compute the momentum density used in Eq. (2.4) only to second order with a separable potential. The first approximation is motivated by the fact that the main objective of the introduction of the t matrix is the removal of the apparent singularity due to the hard core of the potential, and the most important processes involved are large momentum transfer processes (for which ρ_k approaches n_k). The "justification" for using a separable potential to calculate the momentum density is the fact that the momentum density represents a high degree of "averaging" (in the sense that one more average, over momentum states, yields a constant of motion), and consequently should not be as sensitive to the details of the potential as the t matrix itself. These approximations lead to the following equations for the reaction matrix:

$$t_{ij; kl} = v_{ij; kl} + \sum_{mn} v_{ij; mn} \frac{(1-n_m)(1-n_n)}{\epsilon_k + \epsilon_l - \epsilon_m - \epsilon_n} t_{mn; kl}, \quad (2.5)$$

and for the momentum density [to be used in Eq. (2.4)]⁹:

$$\begin{aligned} \rho_q = n_q \left\{ 1 - \sum_{ln} t_{mn; ql} \frac{(1-n_m)(1-n_n)}{(\epsilon_q + \epsilon_l - \epsilon_m - \epsilon_n)^2} t_{mn; (ql)} n_l + \dots \right\} \\ + (1-n_q) \left\{ \sum_{kl} t_{qn; kl} \frac{(1-n_n)}{(\epsilon_k + \epsilon_l - \epsilon_q - \epsilon_n)^2} \right. \\ \left. \times t_{qn; (kl)} n_k n_l + \dots \right\}. \end{aligned} \quad (2.6)$$

To verify this solution, we substitute (2.6) in (2.4):

$$\begin{aligned} \Delta E &= \frac{1}{2} \sum t_{kl; (kl)} n_k n_l - \frac{1}{2} \sum_{kl} t_{kl; (kl)} \\ &\quad \times \left\{ 2 \sum_j t_{mn; jl} \frac{(1-n_m)(1-n_n)}{(\epsilon_q + \epsilon_l - \epsilon_m - \epsilon_n)^2} t_{mn; (jl)} n_j \right\} n_k n_l \\ &\quad + \frac{1}{2} \sum_{km} t_{km; (km)} \left\{ 2 \sum_{jn} t_{mn; jl} \frac{(1-n_m)(1-n_n)}{(\epsilon_j + \epsilon_l - \epsilon_m - \epsilon_n)^2} \right. \\ &\quad \left. \times t_{mn; j} n_j \right\} n_k n_l + O(t^4). \end{aligned} \quad (2.7)$$

Or, changing indices

$$\begin{aligned} \Delta E &= \frac{1}{2} \sum t_{kl; (kl)} n_k n_l \\ &\quad - \frac{1}{2} \sum_{kl} t_{kl; mn} \frac{(1-n_m)(1-n_n)}{(\epsilon_k + \epsilon_l - \epsilon_m - \epsilon_n)^2} t_{mn; (kl)} n_k n_l \\ &\quad \times \left\{ \sum_j [t_{jk; (jk)} + t_{jl; (jl)} - t_{jm; (jm)} - t_{jn; (jn)}] n_j \right\}. \end{aligned} \quad (2.8)$$

⁹ For a derivation of this equation, see MS, Eq. (3.5).

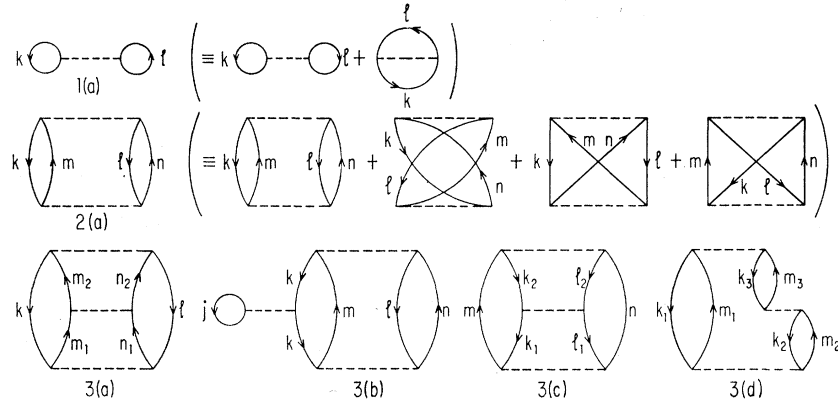


FIG. 1. Low-order diagrams of the Brueckner-Goldstone expansion. The self-energy diagram, 3(b), which is included in the Brueckner approximation through the use of self-consistent energies in the denominators of the K -matrix equation and which is included in the present approximation through the use of momentum densities in the expressions for the single-particle and total energy.

In Eq. (2.8) we note an important feature of this approximation. If we replace each t with v in the second term on the right-hand side, the resulting expression is just the contribution of diagram 3(b) in Fig. 1 to the system energy (with the self-energy insert being included in every possible position in the diagram). Similarly, in an expansion of the Brueckner-Goldstone series in terms of the t matrix [represented diagrammatically by considering each vertex in Fig. 1 a t interaction and eliminating diagrams 2(a) and 3(a)] the contribution of the "self-energy" diagram 3(b) is again just the second term on the right in Eq. (2.8). Thus, the use of the low-order expansion of the momentum density, Eq. (2.6), in the equation for the system energy, Eq. (2.4), picks up the same self-energy diagram that is customarily accounted for by the use of the "self-consistent" energies in the denominator of the K -matrix equation, Eq. (1.1). Clearly, then, this is equivalent to the K -matrix approximation to fourth order.

In order to compute the t matrix with a hard-core potential, the further approximation of BM is employed. In this approximation it is assumed that the difference in energy of the energy denominators can be treated as being independent of the total momentum without incurring appreciable error. Thus

$$t_{k';k} = v_{k';k} + \sum_{k''} \frac{v_{k';k''} t_{k'',k}}{\sum -k''^2/M}, \quad (2.9)$$

with

$$\begin{aligned} \sum &= k'^2/M \text{ holes} \\ &= \begin{cases} = k'^2/M; & k' < p_F \\ = p_F^2/M; & k' \geq p_F \end{cases} \text{ particles.} \end{aligned} \quad (2.10)$$

The approximation now consists of the following system of equations from BM [Eqs. (2.3) through (2.8) and (1.3)]. Where applicable, the modifications in the equations due to the use of the Masterson-Sawada approximation are indicated.

(1) Green's functions:

$$G_{k'}^l(r, r') = \frac{1}{2\pi^2} \int_0^\infty \frac{k''^2 dk'' j_i(k''r) j_i(k''r') f(\bar{P}, k'')}{2[\omega_k - \omega_{k''}]}, \quad (2.11)$$

with the self-consistent denominator $2[\omega_k - \omega_{k''}]$ of BM reducing to $\sum -k''^2/M$ with \sum given by Eq. (2.10). The Pauli step function, averaged over angles of P , $f(\bar{P}, k'')$, is given (as in BG) by

$$\begin{aligned} f(\bar{P}, k'') &= 0 & k''^2 + \frac{1}{4}\bar{P}^2 \leq p_F^2 \\ &= 1 & k''^2 - \frac{1}{2}\bar{P} \geq p_F \\ &= (k''^2 + \frac{1}{4}\bar{P}^2 - p_F^2)/k''\bar{P} & \text{otherwise} \end{aligned} \quad (2.12)$$

with \bar{P} given by (1.5).

(2) Momentum density (not used in BG or BM), Eq. (2.6).

(3) Plane-wave basis functions and Green's functions modified to vanish at the hard-core radius r_c :

$$s_l(kr) = j_l(kr) - j_l(kr_c) G_{k'}^l(r, r_c) / G_{k'}^l(r_c, r_c), \quad (2.13)$$

$$F_{k'}^l(r, r') = G_{k'}^l(r, r') - G_{k'}^l(r, r_c) G_{k'}^l(r_c, r') / G_{k'}^l(r_c, r_c). \quad (2.14)$$

(4) Radial wave functions:

$$\begin{aligned} U_{l\nu}^{J_s}(k, r) &= s_l(kr) \delta_{l\nu} + 4\pi \sum_{l'} \int_{r_c}^\infty r'^2 dr' F_{k'}^{l'}(r, r') \\ &\quad \times V_{\nu l' l}^{J_s}(r') U_{l\nu}^{J_s}(k, r'). \end{aligned} \quad (2.15)$$

The $V_{\nu l' l}^{J_s}(r')$ are the appropriate phenomenological two-body potentials.

(5) t matrices:

$$\begin{aligned} t_{k; k} &= t(k) = \sum_{J_s} \sum_{l=J-1}^{J+1} C_{Jls} \left\{ -j_l^2(kr_c) / G_{k'}^l(r_c, r_c) \right. \\ &\quad \left. + 4\pi \int_{r_c}^\infty r^2 dr s_l(kr) \sum_{l'=J-1}^{J+1} V_{l\nu}^{J_s}(r) U_{l\nu}^{J_s}(k, r) \right\}. \end{aligned} \quad (2.16)$$

(6) Single-particle potential:

$$\begin{aligned} V(p) &= \frac{2}{\pi^2} \left[\int_0^{\frac{1}{2}(p_F - p)} k^2 dk \int_{-1}^1 t_{k; k\rho}(4k^2 + p^2 - 4pk\mu)^{1/2} d\mu \right. \\ &\quad \left. + \int_{\frac{1}{2}(p_F - p)}^{\frac{1}{2}(p_F + p)} k^2 dk \int_{-(p_F^2 - p^2 - 4k^2/4k\rho)}^1 t_{k; k\rho}(4k^2 + p^2 - 4pk\mu)^{1/2} d\mu \right] \\ &\quad \times t_{k; k\rho}(4k^2 + p^2 - 4pk\mu)^{1/2} d\mu \quad p \leq p_F. \end{aligned} \quad (2.17)$$

For $p > p_F$ the first integral vanishes.

(7) Average energy per particle, derived from Eq. (2.4):

$$E_{av} = -\frac{3}{5} \frac{p_F^2}{2M} + \frac{3}{2p_F^2} \int_0^\infty p^2 dp V(p) \rho(p). \quad (2.18)$$

This system of equations, (2.11) through (2.18), differs from the equations of BM in two respects: (1) the use of kinetic energies instead of self-consistent single-particle energies in the denominator of the Green's functions, Eq. (2.11); and, (2) the appearance of the momentum density, Eq. (2.6), in Eqs. (2.17) and (2.18). The Green's functions are the most difficult and time-consuming portion of this computation, and the release of their energy denominators from the self-consistency requirement represents a major simplification. In BG and BM, these energies couple this equation to the output of a previous iteration, and the Green's functions therefore must be computed in each iterative cycle. In the approximation of this paper, with only kinetic energies in the Green's functions and with a second order in t calculation of the momentum density, the system of equations is fully decoupled. The Green's functions can now be calculated once with high precision and tabulated for all future uses of the approximation. Similarly, the momentum density can be calculated once with a hard-shell separable potential and likewise tabulated. The remaining equations are easily solved for a large number of phenomenological potentials, either directly [Eqs. (2.13), (2.14)], by simple iteration [Eq. (2.15)], or by numerical integration [Eqs. (2.16) through (2.18)]. These remaining calculations are even feasible with a desk calculator.

In some of the previous equations one encounters singularities at the Fermi surface, singularities which have been related to the phenomenon of superconductivity. We neglect this effect in these calculations, assuming its quantitative effect on the mean energy to be small. In the earlier approximations, these singularities had negligible effect because the mesh used in the numerical integral of the mean energy per particle avoided the singular point and because the self-consistent single-particle energies for particles outside the Fermi sea were computed "off-energy-shell" in order to include, in an average fashion, a maximum number of higher order terms in the Brueckner-Goldstone expansion. In the derivation of Eq. (2.2), however, MS made no explicit reference to the perturbation series, and thus all momentum densities must be computed "on-energy-shell." Consequently, a principal value integral is indicated, and it can be approximated computationally by a replacement of the form

$$\frac{1}{k'^2 - k''^2} \rightarrow \frac{k'^2 - k''^2}{(k'^2 - k''^2)^2 + \delta^2}, \quad (2.19)$$

where δ^2 is chosen small enough (≈ 5 to 10 MeV) that

the two sides of (2.19) are essentially equal except very near the singularity at $k' = k'' = k_F$.

We might mention at this point several other nuclear matter approximations which are also computationally simpler than those of Brueckner and Gammel¹ and Brueckner and Masterson⁵ and which yield similar saturation properties. These include the separation method of Moszkowski and Scott,¹⁰ the approximations of Puff¹¹ and of Falk and Wilets¹² and the reference spectrum method of Bethe, Brandow, and Petschek.¹³ The separation method simplifies the problem by separating the nuclear potential into short and long-range parts in such a manner that the diagonal elements of the reaction matrix appropriate to the short-range part vanish and the reaction matrix for the long-range part can be approximated by the long-range potential alone. The method leads to explicit formulas for higher order terms, but does not escape from the self-consistency requirements for the single-particle potentials when tensor interactions are considered. Bethe and his collaborators have employed similar techniques to develop a method based on computing a relatively simple "reference" reaction matrix and to derive equations for the difference between this reaction matrix and the nuclear reaction matrix. Again, one needs a "self-consistent" single-particle spectrum, but in their approximation it can be adequately represented by a quadratic expression (effective-mass approximation) with appropriately determined coefficients. This approximation has proved valuable for estimating the effects of higher order terms not usually included in perturbation calculations (e.g., Rajaraman¹⁴ and Petschek¹⁵). The Puff and Falk-Wilets calculations differ from the above in that they are derived from Green's function theory and, in effect, reduce to an approximation very similar to that of Brueckner and Masterson except that the Pauli principle is ignored and the self-consistent energy for particles above the Fermi sea is taken to be the kinetic energy alone. This simplifies, but does not remove, the self-consistency problem; hole energies must still be "self-consistent."¹⁶

It is emphasized that the approximation described in this paper differs from preceding approximations in one important aspect—the replacement of the number operator \hat{n}_k by the density ρ_k and the corresponding approximate cancellation of the self-energy terms. This approximation, represented by Eq. (2.2), was originally derived in a manner independent of pertur-

¹⁰ S. A. Moszkowski and B. L. Scott, *Ann. Phys. (N. Y.)* **11**, 31 (1960); B. L. Scott and S. A. Moszkowski, *Ann. Phys. (N. Y.)* **14**, 107 (1961).

¹¹ R. D. Puff, *Ann. Phys. (N. Y.)* **13**, 317 (1961).

¹² D. S. Falk and L. Wilets, *Phys. Rev.* **124**, 1887 (1961).

¹³ H. A. Bethe, B. H. Brandow, and A. G. Petschek, *Phys. Rev.* **129**, 225 (1963).

¹⁴ R. Rajaraman, *Phys. Rev.* **129**, 265 (1963).

¹⁵ A. G. Petschek, *Phys. Rev.* **133**, B1401 (1964).

¹⁶ See BM for further quantitative discussion of the Moszkowski-Scott approximation and the Puff and Falk-Wilets calculations.

bation theory, but it can be justified and higher order correction terms obtained by perturbation expansions. With the further approximations (2.5) and (2.6), this approximation has the advantage of being relatively easy to calculate because of the decoupling of all of its equations (2.11) through (2.18). Whether or not this approximation is more rapidly convergent than any of the others mentioned has not been determined.

III. NUMERICAL EVALUATION

The approximation suggested above has been evaluated in two different variations, both employing slightly modified versions of the Brueckner-Masterson code.

The most accurate variation was effected most simply. The first iteration of the full system of equations described in that paper was executed as usual, with kinetic energies in the denominators of the Green's functions. Then, on the second (and last) iteration, the propagator in the Green's function was replaced as follows:

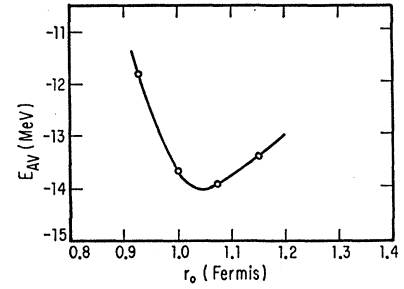
$$\frac{M}{k^2 - k'^2} \rightarrow \frac{M}{k^2 - k'^2} - \frac{M}{(k^2 - k'^2)^2} \times [V_{\text{hole}}(k) - V_{\text{particle}}(k')], \quad (3.1)$$

which, in the BM approximation, is equivalent to

$$\frac{1}{\epsilon_k + \epsilon_l - \epsilon_m - \epsilon_n} - \sum_j \frac{[t_{jk}; (jk) + t_{jl}; (jl) - t_{jm}; (jm) - t_{jn}; (jn)]}{(\epsilon_k + \epsilon_l - \epsilon_m - \epsilon_n)^2}. \quad (3.2)$$

This replacement can easily be shown to lead to Eq. (2.8), which in turn has been shown to be equivalent to using the t matrix given by Eq. (2.5) and the momentum density given by Eq. (2.6) in the equation for the system energy, Eq. (2.4). One variation from the prescription of the previous section was employed for computational simplicity. The previous derivations all specify that principal value integrals be employed in the calculation of the single-particle potentials in (3.1) for particles (with momenta greater than the Fermi momentum) in contrast to the Brueckner-theory calculations in which these potentials are calculated off-energy-shell. Because only differences in single-particle potentials enter into the calculations, and because the curve of the off-energy-shell potential is very nearly parallel to the on-energy-shell potential (see Fig. 4 of Brueckner-Masterson), we have computed the difference in potentials in (2.1) with both hole and particle potentials computed off-energy-shell by the same amount. This procedure avoids the necessity for principal value integrals, and introduces an error in the mean energy per particle which is estimated to be about

FIG. 2. E_{av} versus r_0 for the approximation of this paper, using a modified version of the Brueckner-Masterson code (see text).



0.1 MeV, well within the over-all accuracy of the approximation. In a more rigorous calculation with this approximation, we would use the principal value integral, however.

This calculation yielded the curve of E_{av} versus r_0 shown in Fig. 2. The minimum (equilibrium value) of E_{av} is -14.0 MeV, occurring at an equilibrium spacing of 1.04 F. These results are in remarkably good agreement with those obtained by BG and BM (-15.2 MeV at 1.02 F and -16.9 MeV at 1.00 F, respectively), and with the semiempirical values usually quoted, -15.83 MeV to -17.04 MeV with $r_0 = (1.07 \pm 0.02)$ F. This apparent accuracy, however, is partially a result of fortuitous cancellation of terms for all these calculations, since the perturbation sequence indicates a possible residual error of as much as 5 MeV. For example, the perturbation sequence for the BM calculations was: $+28.8$ (kinetic energy), -61.8 (t matrix), $+17.3$ (first self-energy correction). For the calculation reported in this paper, the first two orders were the same, and the third-order term, the approximation to the density correction, was $+19.0$ MeV.

The results reported above are probably slightly different from those which would be obtained from solving Eqs. (2.11) through (2.18) directly because (1) the momentum density (2.6) was, in effect, computed from the phenomenological hard-core potential instead of being computed directly from a separable, hard-shell potential, and (2) the fourth-order (in t) term from the expansion of Eq. (2.4) was missed. These two factors could lead to a difference in mean binding energy of several MeV, even with an optimum choice of separable potential with which to compute the momentum density.

One attempt was made to analyze the effect of the choice of separable potential for the density calculation (2.6) on the approximation. A Yamaguchi potential without hard shell or hard core¹⁷ was used to calculate ρ_k through second order in t (Fig. 3), and this density was used with the t matrices from the first iteration of BM in Eqs. (2.17) and (2.18) for the system energy. The resulting average energy was -32.1 MeV at $\rho_F = 1.52 \text{ F}^{-3}$. The binding energy was clearly excessive

¹⁷ Y. Yamaguchi, Phys. Rev. **95**, 1629 (1961).

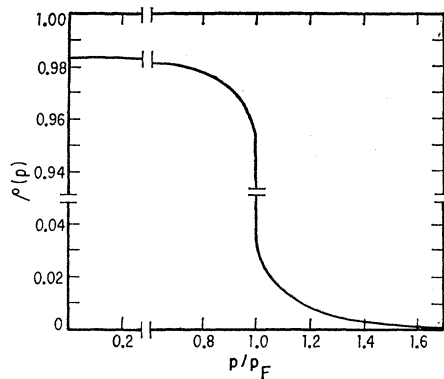


FIG. 3. Momentum density using the Yamaguchi potential. The potential does not have a hard core.

in this approximation using this potential, and no attempt was made to find a minimum. The excessive attraction was a consequence of the high density of states below the Fermi sea allowed by the "soft" potential, and the importance of using a hard core (or hard shell) interaction in the computation of the density is clearly indicated. Indeed, the average density below the Fermi sea with a hard core is about 0.87^{13} versus 0.98 obtained with the Yamaguchi potential. In addition to having a hard core or hard shell, it may also be important that the potential include a reasonable tensor-central force ratio. If so, the density contribution will be most difficult to evaluate. However, since ρ_k represents an average over the t -matrix elements and since the third-order term in (2.8) will include one power of t computed with the full potential, it is probable that just including the core effects will be

sufficient for the type of investigations for which this approximation will be useful. A program is now underway to use an S -state potential with hard shell (the same as that used by Puff¹² and by Falk and Willets,¹³ which is known to give reasonable saturation properties), as well as to modify the BM code for a very high precision (and time-consuming) calculation of the Green's functions. The results will be reported when available.

IV. CONCLUSIONS

Once the density and Green's functions have been calculated and reported, this approximation should be quite useful for a large variety of investigations. As emphasized earlier, it can even be completed on a desk calculator. Among the possible uses one can envision are the evaluation of the effects of various phenomenological potentials, further investigation into the influence of various terms in the potentials and in the perturbation expansion, and any other investigations into the nuclear matter problem where relative comparisons would be informative. The approximation might even be simple enough to use that it could serve as a problem in a graduate course. It would be simple to teach, and its solution should prove quite instructive.

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